

Contents

Preface

xv

I	An Introduction to Quantitative Risk Management	1
1	Risk in Perspective	3
1.1	Risk	3
1.1.1	Risk and Randomness	3
1.1.2	Financial Risk	5
1.1.3	Measurement and Management	6
1.2	A Brief History of Risk Management	8
1.2.1	From Babylon to Wall Street	8
1.2.2	The Road to Regulation	15
1.3	The Regulatory Framework	20
1.3.1	The Basel Framework	20
1.3.2	The Solvency II Framework	25
1.3.3	Criticism of Regulatory Frameworks	28
1.4	Why Manage Financial Risk?	30
1.4.1	A Societal View	30
1.4.2	The Shareholder's View	32
1.5	Quantitative Risk Management	34
1.5.1	The Q in QRM	34
1.5.2	The Nature of the Challenge	35
1.5.3	QRM Beyond Finance	38
2	Basic Concepts in Risk Management	42
2.1	Risk Management for a Financial Firm	42
2.1.1	Assets, Liabilities and the Balance Sheet	42
2.1.2	Risks Faced by a Financial Firm	44
2.1.3	Capital	45
2.2	Modelling Value and Value Change	47
2.2.1	Mapping Risks	47
2.2.2	Valuation Methods	54
2.2.3	Loss Distributions	58
2.3	Risk Measurement	61
2.3.1	Approaches to Risk Measurement	61
2.3.2	Value-at-Risk	64
2.3.3	VaR in Risk Capital Calculations	67

2.3.4	Other Risk Measures Based on Loss Distributions	69
2.3.5	Coherent and Convex Risk Measures	72
3	Empirical Properties of Financial Data	79
3.1	Stylized Facts of Financial Return Series	79
3.1.1	Volatility Clustering	80
3.1.2	Non-normality and Heavy Tails	85
3.1.3	Longer-Interval Return Series	87
3.2	Multivariate Stylized Facts	88
3.2.1	Correlation between Series	88
3.2.2	Tail Dependence	90
II	Methodology	95
4	Financial Time Series	97
4.1	Fundamentals of Time Series Analysis	98
4.1.1	Basic Definitions	98
4.1.2	ARMA Processes	100
4.1.3	Analysis in the Time Domain	105
4.1.4	Statistical Analysis of Time Series	107
4.1.5	Prediction	109
4.2	GARCH Models for Changing Volatility	112
4.2.1	ARCH Processes	112
4.2.2	GARCH Processes	118
4.2.3	Simple Extensions of the GARCH Model	121
4.2.4	Fitting GARCH Models to Data	123
4.2.5	Volatility Forecasting and Risk Measure Estimation	129
5	Extreme Value Theory	135
5.1	Maxima	135
5.1.1	Generalized Extreme Value Distribution	136
5.1.2	Maximum Domains of Attraction	139
5.1.3	Maxima of Strictly Stationary Time Series	141
5.1.4	The Block Maxima Method	142
5.2	Threshold Exceedances	146
5.2.1	Generalized Pareto Distribution	147
5.2.2	Modelling Excess Losses	149
5.2.3	Modelling Tails and Measures of Tail Risk	154
5.2.4	The Hill Method	157
5.2.5	Simulation Study of EVT Quantile Estimators	161
5.2.6	Conditional EVT for Financial Time Series	162
5.3	Point Process Models	164
5.3.1	Threshold Exceedances for Strict White Noise	164
5.3.2	The POT Model	166
6	Multivariate Models	173
6.1	Basics of Multivariate Modelling	174
6.1.1	Random Vectors and Their Distributions	174
6.1.2	Standard Estimators of Covariance and Correlation	176
6.1.3	The Multivariate Normal Distribution	178
6.1.4	Testing Multivariate Normality	180

6.2	Normal Mixture Distributions	183
6.2.1	Normal Variance Mixtures	183
6.2.2	Normal Mean–Variance Mixtures	187
6.2.3	Generalized Hyperbolic Distributions	188
6.2.4	Empirical Examples	191
6.3	Spherical and Elliptical Distributions	196
6.3.1	Spherical Distributions	196
6.3.2	Elliptical Distributions	200
6.3.3	Properties of Elliptical Distributions	202
6.3.4	Estimating Dispersion and Correlation	203
6.4	Dimension-Reduction Techniques	206
6.4.1	Factor Models	206
6.4.2	Statistical Estimation Strategies	208
6.4.3	Estimating Macroeconomic Factor Models	210
6.4.4	Estimating Fundamental Factor Models	213
6.4.5	Principal Component Analysis	214
7	Copulas and Dependence	220
7.1	Copulas	220
7.1.1	Basic Properties	221
7.1.2	Examples of Copulas	225
7.1.3	Meta Distributions	229
7.1.4	Simulation of Copulas and Meta Distributions	229
7.1.5	Further Properties of Copulas	232
7.2	Dependence Concepts and Measures	235
7.2.1	Perfect Dependence	236
7.2.2	Linear Correlation	238
7.2.3	Rank Correlation	243
7.2.4	Coefficients of Tail Dependence	247
7.3	Normal Mixture Copulas	249
7.3.1	Tail Dependence	249
7.3.2	Rank Correlations	253
7.3.3	Skewed Normal Mixture Copulas	256
7.3.4	Grouped Normal Mixture Copulas	257
7.4	Archimedean Copulas	259
7.4.1	Bivariate Archimedean Copulas	259
7.4.2	Multivariate Archimedean Copulas	261
7.5	Fitting Copulas to Data	265
7.5.1	Method-of-Moments Using Rank Correlation	266
7.5.2	Forming a Pseudo-sample from the Copula	269
7.5.3	Maximum Likelihood Estimation	270
8	Aggregate Risk	275
8.1	Coherent and Convex Risk Measures	275
8.1.1	Risk Measures and Acceptance Sets	276
8.1.2	Dual Representation of Convex Measures of Risk	280
8.1.3	Examples of Dual Representations	283
8.2	Law-Invariant Coherent Risk Measures	286
8.2.1	Distortion Risk Measures	286
8.2.2	The Expectile Risk Measure	290

8.3	Risk Measures for Linear Portfolios	293
8.3.1	Coherent Risk Measures as Stress Tests	293
8.3.2	Elliptically Distributed Risk Factors	295
8.3.3	Other Risk Factor Distributions	297
8.4	Risk Aggregation	299
8.4.1	Aggregation Based on Loss Distributions	300
8.4.2	Aggregation Based on Stressing Risk Factors	302
8.4.3	Modular versus Fully Integrated Aggregation Approaches	303
8.4.4	Risk Aggregation and Fréchet Problems	305
8.5	Capital Allocation	315
8.5.1	The Allocation Problem	315
8.5.2	The Euler Principle and Examples	316
8.5.3	Economic Properties of the Euler Principle	320
III	Applications	323
9	Market Risk	325
9.1	Risk Factors and Mapping	325
9.1.1	The Loss Operator	326
9.1.2	Delta and Delta–Gamma Approximations	327
9.1.3	Mapping Bond Portfolios	329
9.1.4	Factor Models for Bond Portfolios	332
9.2	Market Risk Measurement	338
9.2.1	Conditional and Unconditional Loss Distributions	339
9.2.2	Variance–Covariance Method	340
9.2.3	Historical Simulation	342
9.2.4	Dynamic Historical Simulation	343
9.2.5	Monte Carlo	346
9.2.6	Estimating Risk Measures	347
9.2.7	Losses over Several Periods and Scaling	349
9.3	Backtesting	351
9.3.1	Violation-Based Tests for VaR	352
9.3.2	Violation-Based Tests for Expected Shortfall	354
9.3.3	Elicitability and Comparison of Risk Measure Estimates	355
9.3.4	Empirical Comparison of Methods Using Backtesting Concepts	358
9.3.5	Backtesting the Predictive Distribution	363
10	Credit Risk	366
10.1	Credit-Risky Instruments	367
10.1.1	Loans	367
10.1.2	Bonds	368
10.1.3	Derivative Contracts Subject to Counterparty Risk	369
10.1.4	Credit Default Swaps and Related Credit Derivatives	370
10.1.5	PD, LGD and EAD	372
10.2	Measuring Credit Quality	374
10.2.1	Credit Rating Migration	374
10.2.2	Rating Transitions as a Markov Chain	376
10.3	Structural Models of Default	380
10.3.1	The Merton Model	380
10.3.2	Pricing in Merton's Model	381
10.3.3	Structural Models in Practice: EDF and DD	386
10.3.4	Credit-Migration Models Revisited	389

10.4	Bond and CDS Pricing in Hazard Rate Models	391
10.4.1	Hazard Rate Models	391
10.4.2	Risk-Neutral Pricing Revisited	394
10.4.3	Bond Pricing	399
10.4.4	CDS Pricing	401
10.4.5	P versus Q : Empirical Results	404
10.5	Pricing with Stochastic Hazard Rates	406
10.5.1	Doubly Stochastic Random Times	406
10.5.2	Pricing Formulas	411
10.5.3	Applications	413
10.6	Affine Models	416
10.6.1	Basic Results	417
10.6.2	The CIR Square-Root Diffusion	418
10.6.3	Extensions	420
11	Portfolio Credit Risk Management	425
11.1	Threshold Models	426
11.1.1	Notation for One-Period Portfolio Models	426
11.1.2	Threshold Models and Copulas	428
11.1.3	Gaussian Threshold Models	430
11.1.4	Models Based on Alternative Copulas	431
11.1.5	Model Risk Issues	433
11.2	Mixture Models	436
11.2.1	Bernoulli Mixture Models	436
11.2.2	One-Factor Bernoulli Mixture Models	437
11.2.3	Recovery Risk in Mixture Models	440
11.2.4	Threshold Models as Mixture Models	441
11.2.5	Poisson Mixture Models and CreditRisk ⁺	444
11.3	Asymptotics for Large Portfolios	449
11.3.1	Exchangeable Models	450
11.3.2	General Results	452
11.3.3	The Basel IRB Formula	455
11.4	Monte Carlo Methods	457
11.4.1	Basics of Importance Sampling	457
11.4.2	Application to Bernoulli Mixture Models	460
11.5	Statistical Inference in Portfolio Credit Models	464
11.5.1	Factor Modelling in Industry Threshold Models	465
11.5.2	Estimation of Bernoulli Mixture Models	466
11.5.3	Mixture Models as GLMMs	470
11.5.4	A One-Factor Model with Rating Effect	472
12	Portfolio Credit Derivatives	476
12.1	Credit Portfolio Products	476
12.1.1	Collateralized Debt Obligations	477
12.1.2	Credit Indices and Index Derivatives	481
12.1.3	Basic Pricing Relationships for Index Swaps and CDOs	484
12.2	Copula Models	487
12.2.1	Definition and Properties	487
12.2.2	Examples	489
12.3	Pricing of Index Derivatives in Factor Copula Models	491
12.3.1	Analytics	491
12.3.2	Correlation Skews	494
12.3.3	The Implied Copula Approach	497

13 Operational Risk and Insurance Analytics	503
13.1 Operational Risk in Perspective	503
13.1.1 An Important Risk Class	503
13.1.2 The Elementary Approaches	505
13.1.3 Advanced Measurement Approaches	506
13.1.4 Operational Loss Data	509
13.2 Elements of Insurance Analytics	512
13.2.1 The Case for Actuarial Methodology	512
13.2.2 The Total Loss Amount	513
13.2.3 Approximations and Panjer Recursion	518
13.2.4 Poisson Mixtures	524
13.2.5 Tails of Aggregate Loss Distributions	525
13.2.6 The Homogeneous Poisson Process	526
13.2.7 Processes Related to the Poisson Process	529
 IV Special Topics	 537
14 Multivariate Time Series	539
14.1 Fundamentals of Multivariate Time Series	539
14.1.1 Basic Definitions	539
14.1.2 Analysis in the Time Domain	541
14.1.3 Multivariate ARMA Processes	542
14.2 Multivariate GARCH Processes	545
14.2.1 General Structure of Models	545
14.2.2 Models for Conditional Correlation	547
14.2.3 Models for Conditional Covariance	550
14.2.4 Fitting Multivariate GARCH Models	553
14.2.5 Dimension Reduction in MGARCH	554
14.2.6 MGARCH and Conditional Risk Measurement	557
 15 Advanced Topics in Multivariate Modelling	 559
15.1 Normal Mixture and Elliptical Distributions	559
15.1.1 Estimation of Generalized Hyperbolic Distributions	559
15.1.2 Testing for Elliptical Symmetry	562
15.2 Advanced Archimedean Copula Models	566
15.2.1 Characterization of Archimedean Copulas	566
15.2.2 Non-exchangeable Archimedean Copulas	568
 16 Advanced Topics in Extreme Value Theory	 572
16.1 Tails of Specific Models	572
16.1.1 Domain of Attraction of the Fréchet Distribution	572
16.1.2 Domain of Attraction of the Gumbel Distribution	573
16.1.3 Mixture Models	574
16.2 Self-exciting Models for Extremes	577
16.2.1 Self-exciting Processes	578
16.2.2 A Self-exciting POT Model	580
16.3 Multivariate Maxima	583
16.3.1 Multivariate Extreme Value Copulas	583
16.3.2 Copulas for Multivariate Minima	586
16.3.3 Copula Domains of Attraction	586
16.3.4 Modelling Multivariate Block Maxima	589

16.4	Multivariate Threshold Exceedances	591
16.4.1	Threshold Models Using EV Copulas	591
16.4.2	Fitting a Multivariate Tail Model	592
16.4.3	Threshold Copulas and Their Limits	594
17	Dynamic Portfolio Credit Risk Models and Counterparty Risk	599
17.1	Dynamic Portfolio Credit Risk Models	599
17.1.1	Why Dynamic Models of Portfolio Credit Risk?	599
17.1.2	Classes of Reduced-Form Models of Portfolio Credit Risk	600
17.2	Counterparty Credit Risk Management	603
17.2.1	Uncollateralized Value Adjustments for a CDS	604
17.2.2	Collateralized Value Adjustments for a CDS	609
17.3	Conditionally Independent Default Times	612
17.3.1	Definition and Mathematical Properties	612
17.3.2	Examples and Applications	618
17.3.3	Credit Value Adjustments	622
17.4	Credit Risk Models with Incomplete Information	625
17.4.1	Credit Risk and Incomplete Information	625
17.4.2	Pure Default Information	628
17.4.3	Additional Information	633
17.4.4	Collateralized Credit Value Adjustments and Contagion Effects	637
Appendix		641
A.1	Miscellaneous Definitions and Results	641
A.1.1	Type of Distribution	641
A.1.2	Generalized Inverses and Quantiles	641
A.1.3	Distributional Transform	643
A.1.4	Karamata's Theorem	644
A.1.5	Supporting and Separating Hyperplane Theorems	644
A.2	Probability Distributions	644
A.2.1	Beta	645
A.2.2	Exponential	645
A.2.3	F	645
A.2.4	Gamma	645
A.2.5	Generalized Inverse Gaussian	646
A.2.6	Inverse Gamma	646
A.2.7	Negative Binomial	646
A.2.8	Pareto	647
A.2.9	Stable	647
A.3	Likelihood Inference	647
A.3.1	Maximum Likelihood Estimators	648
A.3.2	Asymptotic Results: Scalar Parameter	648
A.3.3	Asymptotic Results: Vector of Parameters	649
A.3.4	Wald Test and Confidence Intervals	649
A.3.5	Likelihood Ratio Test and Confidence Intervals	650
A.3.6	Akaike Information Criterion	650
References		652
Index		687